History-Dependent Capital Taxation

Abstract

This paper provides an alternative explanation for a nonzero tax rate on capital, reexamining Ramsey's(1927) rule. Due to a lack of commitment power from government, households form adaptive expectation on the capital tax rate. The equilibrium capital tax rate is thus history-dependent with a balanced-budget requirement. The investment decision combines income and substitution effects, and the U.S. states differ on investment sensitivity to capital tax rates. This paper first provides empirical findings on investment sensitivity for each state, and then accounts for both the level and pattern of historic capital tax rates at the state level. The simulated results qualitatively match the empirical evidence observed across 50 states.

1 Introduction

Ramsey's (1927) seminal contribution on zero capital taxation states that in order to ensure a bounded future implicit consumption tax and avoid capital accumulation distortion, it is optimal to levy zero taxation on capital investment in the long run in an infinitely-lived household model. Further studies including Chamley (1986) and Judd (1985) validated this result under different economic environments.

Based on these, a lot of literature was extended in various directions and gave stories that optimal capital tax should not be zero, including different discount factors across individuals (Diamond and Spinnewijn, 2011); financial market failures (Glenn and Judd,1986; Aiyagari, 1995); life-cycle models (Erosa and Gervais, 2002; Garriga, 2003); nonseparable utility (Kuhn and Koehne, 2013) and so on.

After observing data on capital tax rates all over the world, the gap between tax rates in reality and that suggested by Ramsey's theory is evident: some countries impose relatively high taxation on capital, whereas some other countries levy no tax on capital.

In the US, apart from federal capital taxation, each state imposes its own tax rate of capital. Thus I continue to check historical capital taxation across 50 states in the US, and found that capital tax rates are different across states in terms of both levels and historical patterns, which can be decreasing, increasing or oscillating. Furthermore, the ratio of capital to personal income(inflation-adjusted) shows a fluctuating but convergent pattern in most states.

This paper provides an explanation to capture these empirical facts across states in the US, based on observations above.

The time lag between individuals' investment and return realizations leads to uncertainty about future returns when households invest. If government has no commitment power on capital taxation, households form their own expectations on future tax rate and invest accordingly. In the next period, capital tax rate is realized based on existing capital stock, and households update their belief according to new information.

Economists have been debating over the assumptions of rational expectation and adaptive expectation when it comes to study of economic behavior. The rational expectation hypothesis is argued to be a possible source of the Lucas Critique, and is thus supported and applied widely. However,this possibility does not validate rational expectations due to a lack of empirical support, as suggested by Chow (2011). Furthermore, Chow (2011) and Chow (1988) presented strong statistical and econometric evidence for adaptive expectation. Logical argument is also provided for using adaptive expectation as a better proxy for psychological expectation. Hence, households in my model update their belief using adaptive expectations. The expected tax rate is a weighted sum of past information with geometrically declining weights with respect to time.

All but one U.S states are required to expend no more than the revenue they can raise[1]. States start with different initial beliefs on tax rates, which lead to different historical patterns of tax rates. With low initial expectation on capital tax rate, households invest a lot and increases the tax base, government only needs to levy low tax rate, which confirms households' initial belief; while with high initial capital tax expectation, households reduce investment sufficiently, which forces government to tax heavily on capital return to meet government budget, thus government can do nothing to revert households' belief back to low level, and get "stuck" in the high tax equilibrium. With balanced budget requirement, the government cannot borrow to alter households' belief; nor can this be achieved with no commitment power from the government. Thus, the existence of multiple equilibria is possible, suggesting that given different levels of initial capital stock or initial expectation, each state will end up at different steady states.

Moreover, the equilibrium with higher capital tax rate is associated with lower capital stock, which matches the findings of empirical work that the capital level as well as investment is negatively correlated with capital tax rate (Knight, 2000).

The historical path of capital tax rates is also determined by the elasticity of investment to changes in tax rates. I empirically estimate the investment sensitivity to tax rates for each state. This investment decision rule is a combination of substitution and income effects, which offset with each other: with an increasing expected capital tax rate, the substitution effect decreases capital investment, while income effect increases capital investment. As states differ in industry structure, productivity level, education level and degree of economic inequality, it's natural to observe different investment behavior empirically. Each state's specific investment curve and starting belief characterize the pattern of tax rates evolution. The simulated sequence of tax rates qualitatively fits the observed data.

[1] The National Conference of State Legislatures (NCSL) has traditionally reported that 49 states must balance their budgets, with Vermont being the exception.

The structure of this paper is as follows: Section 2 introduces dataset and provides empirical findings, and Section 3 introduces the empirical model. Section 4 summarizes the result and Section 5 introduces the potential structural model. Lastly, Section 6 concludes.

2 Data

2.1 Average Capital Gains Tax Rates

The officially available data on capital taxation includes marginal capital gains tax rates, brackets and so on. These information have been used to calculate effective capital tax rates. Pomerleau (2013) shows a wide range of effective rates across states.

In most theoretical models, return from capital investment is taxed proportionally and thus the tax rate is simplified as an average tax rate. However, insufficient empirical work has been done to obtain average capital tax rates in the US or in the states. In order to be consistent with theoretical models, I obtained my own series of average capital tax rates for each state [2] from 1958 to 2008.

From US Census Bureau, I first summed up three sources of revenue to account for capital tax revenue: property tax, corporate net income tax, death and gift tax [3]. And I used the data "dividends, interest and rent" for taxable capital income. Then I divided total capital tax revenue by taxable capital income to get the average capital tax rates.

The calculated tax rates range from 0 to 0.25, and most of them fall into the range of 0 to 0.1. States start with different initial values of rates in 1958, which I summarize in Appendix I. The paths of historical rates can be categorized into three patterns: decreasing (e.g. Texas), oscillating (e.g. North Dakota), and increasing (e.g. New Hampshire). The figures of these three states' patterns are displayed in Figure 1.

[2] I included District of Columbia for analysis as well. And there exist some errors in data for several years of Alaska, which is a fiscal and geographic outlier in the US. The observations of Alaska could thus be dropped.

[3] Death and Gift tax is the tax imposed on transfer of property at death, in contemplation of death, or as a gift.

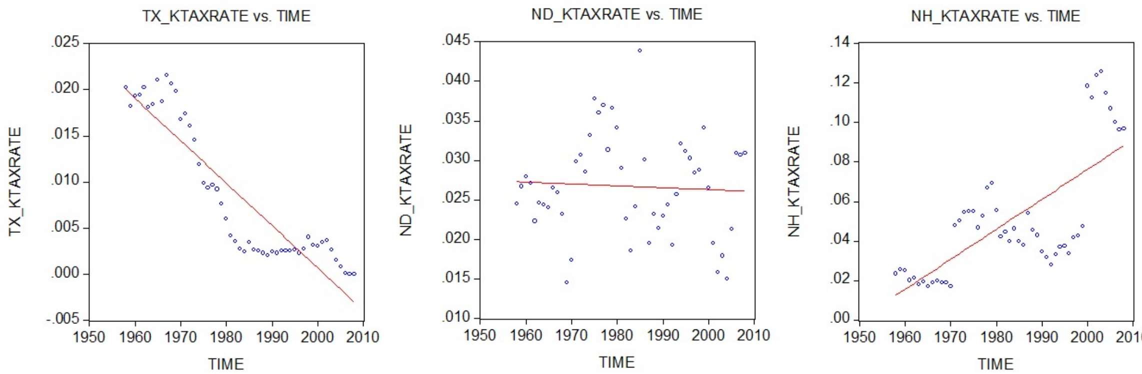

Figure 1: Capital tax rates in Texas, North Dakota and New Hampshire

2.2 Detrended Capital Stocks

To estimate investment decision empirically, I obtain the data on capital stock from the database created by Garofalo and Yamarik(2002) and Yamarik(2012). As capital grows over time in each state, I first detrend capital by dividing the level of capital in each state by the aggregate level of capital in the United States in each year. The detrended capital stock thus represents the percentage of each state's capital level in the US. And these data will be used for empirical regression, with the details presented in section 3.

The correlation between detrended capital and capital tax rates is either negative or positive, which implies different investment behaviors across states. This will be discussed in next section.

The patterns of detrended capital of the three states mentioned in last section are displayed in Figure 2.

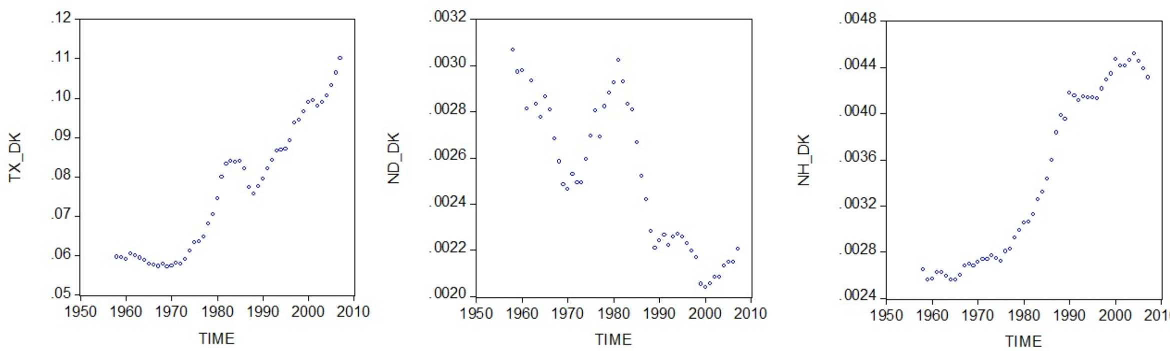

Figure 2: Detrended capital stock in Texas, North Dakota and New Hampshire

2.3 Government spending

State government is not encouraged to borrow to meet their expenditure, so the major source of raising revenue is from taxes.

In a growing economy, both capital tax revenue and state government expenditure show an upward sloping trend in each state. To detrend government spending and obtain the portion of state government expenditure covered by capital taxation, I divided capital tax revenue by total government expenditure. This is consistent with the way capital is detrended.

3 Empirical Model

3.1 Investment Sensitivity

The tax sensitivity of investment has important implications for analyzing historical pattern of capital taxation. The investment decision is influenced by a combination of income and substitution effects, and the total effect can vary at different values of the capital tax rate. With an increase in expected capital tax rate, the expected net return decreases and households reduce investment, which characterizes the substitution effect. A higher expected capital tax rate also reduces expected income next period, and in order to ensure a certain level of consumption, households increase investment. This income effect offsets the substitution effect.

Households' responses to changes in taxation at different levels of rates generate an investment decision curve over the whole range of capital tax rates. Each state has its own feature of industrial structure, productivity level, education level and degree of economic inequality, which altogether produce a specific investment decision curve for each state.

As argued in Young (1988), many factors influence the decision on investment, with some of them difficult to be quantified. In this paper, the effects of tax rates are isolated to be analyzed.

To estimate the elasticity of investment for each state, I run a regression of the capital level on a polynomial of tax rates as in Equation (1), which is based on the Hartman (1985) model. I use annual data on detrended capital and contemporary capital tax rate [4] for each state. And the regression result can be linear, quadratic or cubic, the most significant one of which is chosen as the investment decision curve for each state, so c_2, c_3 can be zero [5].

$$dk_t = c_0 + c_1\theta_t + c_2\theta_t^2 + c_3\theta_t^3 + \epsilon_t \tag{1}$$

Estimation results indicate that states are distributed approximately equally into four main patterns of investment behavior: decreasing, increasing, U-shaped and inverse U-shaped, with a few left maintaining a polynomial of cubic

[4] There is no significant difference in regression results when lagged capital tax rates are used as explanatory variables, from a randomly selected sample of states.

[5] Only a few states have regression results with a polynomial of degree higher than 3.

or even higher degree. Estimation results of representative state in each pattern are displayed in Table 1.

Table 1: Tax Sensitivity Estimation of Representative States

Dependent variable: DetrendedCapital

Sensitivity Patterns (State)	Decreasing (Alabama)	Increasing (Iowa)	U-shape (Virginia)	Inverse U-shape (New Mexico)	Cubic (Illinois)
c_0	0.013***	0.008***	0.025***	0.003***	0.097***
	(0.0002)	(0.0009)	(0.0005)	(0.0005)	(0.0063)
c_1	-0.026***	0.119***	-0.228***	0.083***	-5.387***
	(0.005)	(0.037)	(0.0408)	(0.0282)	(1.0480)
c_2	0	0	1.382**	-0.927**	185.828***
			(0.6219)	(0.34860)	(43.1458)
c_3	0	0	0	0	-1894.047***
					(508.7329)

Note: These are least squares estimates of the parameters in Eq. (1).

Robust standard errors in parentheses. ***p<0.01, **p<0.05, *p<0.1.

And the corresponding investment curves for those representative states are depicted in Figure 3, where I denote t as capital tax rate and dkfcst as the estimation forecast value of detrended capital.

Different curves of investment decision imply different combinations of substitution and income effects, with the main patterns summarized below:

Case 1 *Decreasing pattern*

Substitution effect dominates income effect in the whole range of tax rates. As the expected capital tax rate increases, the return of investment decreases and thus investment decreases.

Case 2 *Increasing pattern*

The income effect dominates the substitution effect in the whole range of tax rates. As the expected capital tax rate increases, income from investment decreases and thus investment increases to compensate for the loss in expected income.

Case 3 *U-shaped pattern*

The substitution effect dominates in the lower range of tax rates while the income effect dominates in the higher range. As the expected capital tax rate rises in the higher range, low income further decreases, which households are very sensitive to, and investment increases accordingly.

Case 4 *Inverse U-shaped pattern*

The income effect dominates in the lower range of tax rates while the substitution effect dominates in the higher range. Households are more sensitive to tax rate changes at higher level of capital tax rates; while at lower level of tax rates, households care more about the income loss.

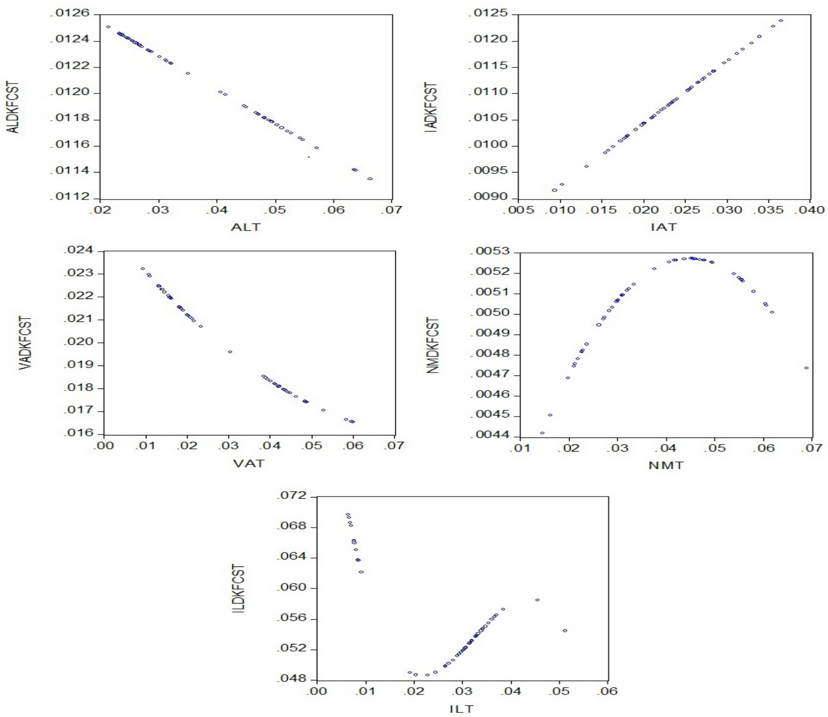

Figure 3: Investment Decision

States differ in terms of investment patterns, which are summarized in Appendix II. Further analysis to account for their patterns are in Section 5.

3.2 Adaptive Expectation

Economists have been studying the hypotheses of Rational Expectation (RE) and Adaptive Expectation (AE), by testing for the empirical validity of each. Campbell and Shiller (1987), Poterba and Summers (1987), Fama and French (1988) and West (1988) realized the inconsistency with data from the assumption of Rational Expectation in present-value models. Moreover, Chow (1988) found that by replacing Rational Expectation hypothesis with Adaptive Expectation, the performance of present-value models in explaining data improves.

Chow (2011) summarizes how these two competing hypotheses can be effectively assumed and provide further econometric support for Adaptive Expectation. Though Rational Expectation has been long accepted for its potential to serve as a source of the Lucas critique, this alone does not rationalize the use of Rational Expectation as the empirical economic hypothesis over Adaptive Expectation, with insufficient evidence supporting Rational Expectation.

Moreover, a large body of research has been testing on the RE hypothesis using survey data of inflation expectations, including Bonham and Cohen (2001), Bonham and Dacy (1991) and Croushore (1997). All of them failed to empirically justify the RE assumption. Similarly, literature such as Frankel and Froot (1987b, 1990a), Froot (1989), Friedman (1990) and Jeong and Maddala (1996) applied the survey data of interest rate forecasts from foreign exchange markets and found that the traders' behaviors display behavioral instead of rational patterns. Thus, these findings also rejected the RE hypothesis and motivated economists to search for alternative models to match the survey data. Markiewicz and Pick (2013) is one of these contributions to support the approach of adaptive learning.

Furthermore, a brief observation of the patterns of historical capital tax rates (as shown in Figure 1) suggests the use of Adaptive Expectation hypothesis. Rather than jumping into the steady state equilibrium immediately which is implied by Rational Expectation hypothesis, tax rates gradually converge or oscillate around.

Hence, I assume Adaptive Expectation in what follows, which also makes logical sense as households form their expectations by averaging past information with geometrically declining weights.

Assumption: Adaptive Expectation Denote θ_t^e as the expected tax rate for period t, and θ_t as the realized capital tax rate set by the government at period t, households form expectation according to:

$$\theta_t^e = \lambda\theta_{t-1} + (1-\lambda)\theta_{t-1}^e, 0 \leq \lambda \leq 1 \tag{2}$$

Households update their belief on capital tax rates using newly realized tax rates weighted λ. When λ=1, households fully utilize new information and believe that government will set the same capital tax policy next year. When λ=0, households insist on their initial belief and consider the change in realized capital tax rate merely as a perturbation. A higher λ implies more weight on new information.

3.3 The Model

3.3.1 Households

In this section, households in State i invest according to a reduced form investment function [6], denoted by

$$k_t = f_i(\theta_t^e) \tag{3}$$

[6] The structural model is presented in Section 5

which is empirically estimated in Section 3.1.
Starting with an initial belief θ_0^e, capital level next period is determined and capital tax rate is realized. Households update their expected tax rate according to Adaptive Expectation Assumption applying new information obtained.

3.3.2 Government

Investment is an intertemporal decision, but government lacks commitment power setting tax policy. Suppose government announces zero capital tax for next year, households who believe it will invest largely for the high return. If faced with a positive spending shock when it comes to next year, government has an incentive to deviate by setting a slightly higher than zero capital tax rate on a large capital stock, in order to acquire revenue and meet the budget requirement. Foreseeing this, commitment from government is not credible to households, which motivates households to form their own beliefs on policy.
Government collects revenue from tax collection. State government is faced with State Balanced-Budget Provision, and cannot borrow to cover expenditure.

Assuming Cobb-Douglas production function, the return from capital is $r_t = A\alpha k_t^{\alpha-1}N^{1-\alpha} - \delta$. Thus the balanced budget equation of government is[7]

$$G_t = k_t r_t \theta_t \tag{4}$$

Government determines capital tax rate each period from this equation.

3.4 Patterns of Tax Evolution

This section describes how different patterns of historical capital tax rates are generated.
To simplify the analysis using figures, I assume λ=1 in equation of Adaptive Expectation.

$$\theta_t^e = \theta_{t-1} \tag{5}$$

And I fix government spending level constant for better illustration, so curve (represented by Equation (3)) does not shift around over time.

Investment decision rule (Equation 2), government balanced budget (Equation 3) and adaptive expectation (Equation 4) simultaneously determine a path of capital tax rates, given an initial expected capital tax rate.

Many possible patterns can be resulted in, which are summarized below.

Pattern 1

[7] I do not include other sources of tax revenue, as the data I use for simulation is the percentage of government expenditure collected from capital tax revenue. Results still hold qualitatively in an alternative setting with other taxes included.

Figure 4 depicts the case with decreasing investment decision curve. Starting from a low initial expected capital tax rate for period 1 at θ_1^e, households invest and capital level is k_1 at period 1, determined by the investment curve. Given k_1, government chooses capital tax rate at θ_1, determined by government revenue curve. Then households update their belief on capital tax rate for period 2 by $\theta_2^e = \theta_1$. With capital tax rate expected at θ_2^e, households invest up to capital level at k_1. Capital tax rates increase monotonely and converge to a stable steady state equilibrium with positive tax rate. This sequence is summarized as pattern 1 in the right graph.

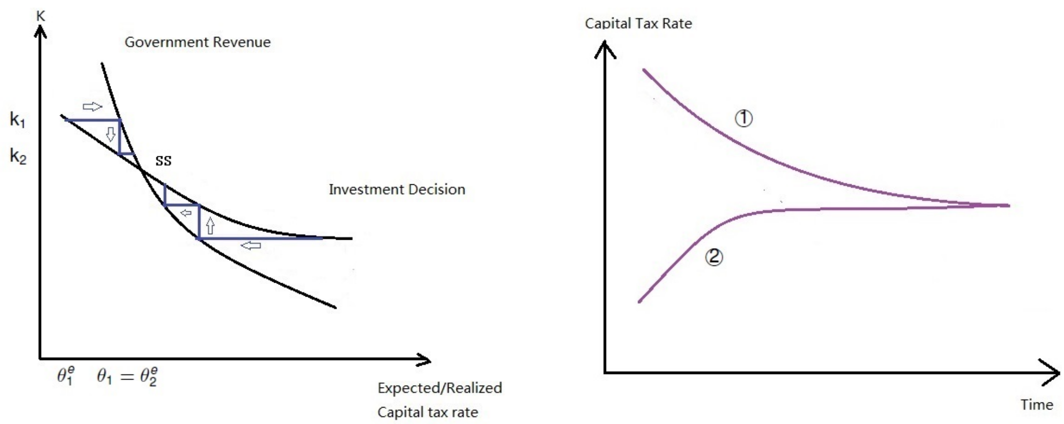

Figure 4: Pattern 1

Similarly, starting from a high initial expected capital tax rate, tax rates monotonely decrease over time and converge to the same stable steady state equilibrium.

Evidently, the Ramsey result does not hold here, as the capital tax rates converge to a positive value rather than zero. With limited commitment power, government cannot alter households' expectation by announcement. Furthermore, with balanced budget requirement, government cannot borrow to set a low rate permanently to enforce a low belief. Hence, households invest according to their initial belief, which is reinforced gradually by government action until the equilibrium is reached.

Moreover, initial belief held by the households matters for the pattern of convergence. A low initial belief gives rise to an increasing pattern, whereas a high initial belief produces a decreasing pattern.

Pattern 2

Here is another case with decreasing investment decision curve as depicted in Figure 5, which is however more steeply sloped than that in Pattern 1.

Different from previous case, if households start with a relatively low expected capital tax rate, the economy converges to a zero capital tax rate, which is consistent with the Ramsey's result. If starting from a relatively high rate, however, the tax rates diverge. Apparently, there does not exist any stable steady state equilibrium in this case.

A more steeply sloped investment curve suggests more elastic response to tax rates by households. At low values of capital tax rates, a reduction in tax rates significantly increases investment, which allows government to further reduce tax rates, and ultimately leads to the convergence to zero tax rate. This is beneficial to the economy, with lower degree of distortion and higher level of capital stock.

At high values of capital tax rates, however, an increase in tax rate reduces investment greatly due to higher sensitivity to tax changes. With big drops in the capital base, the government has to further increase tax rates to meet the budget. This is devastating to the economy with escalating capital tax rates over time.

Initial belief is also crucial here: a low starting belief combined with a sensitive investment curve leads to a decreasing capital tax rate to zero; while a high starting belief combined with the same investment curve "traps" the government in this worsening situation and collapses the economy.

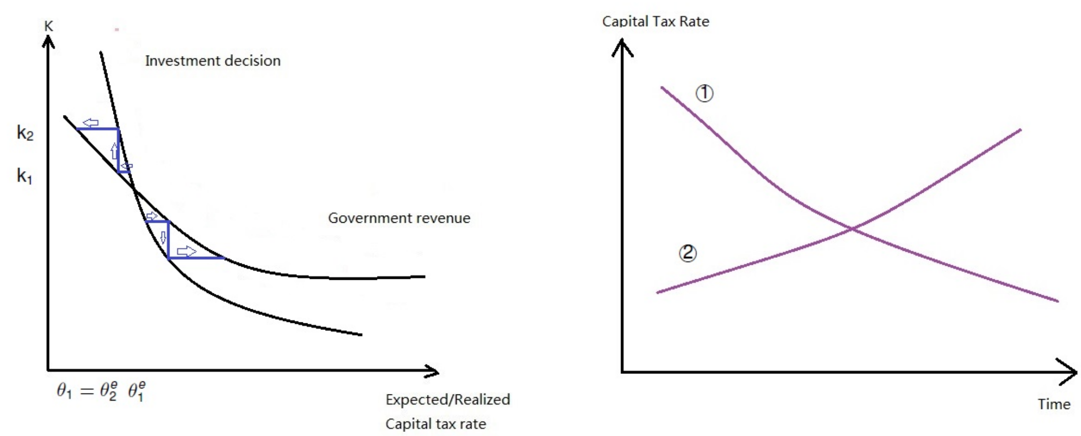

Figure 5: Pattern 2

Pattern 3

Figure 6 shows the case with U-shaped investment decision curve. Starting from any expected rate, tax rates oscillate until reaching the stable steady state equilibrium and stay there. The tax rate at the steady state is positive.

This deviates from Ramsey's rule due to a less elastic investment decision. In the range of high tax rates, as investment decision dominates, households invest a lot and bring down the tax rate to the low range of tax rates. Therefore, no matter where the economy starts, it ends up at the same steady state equilibrium.

Similarly, the specific pattern of tax rates path depends on the initial belief.

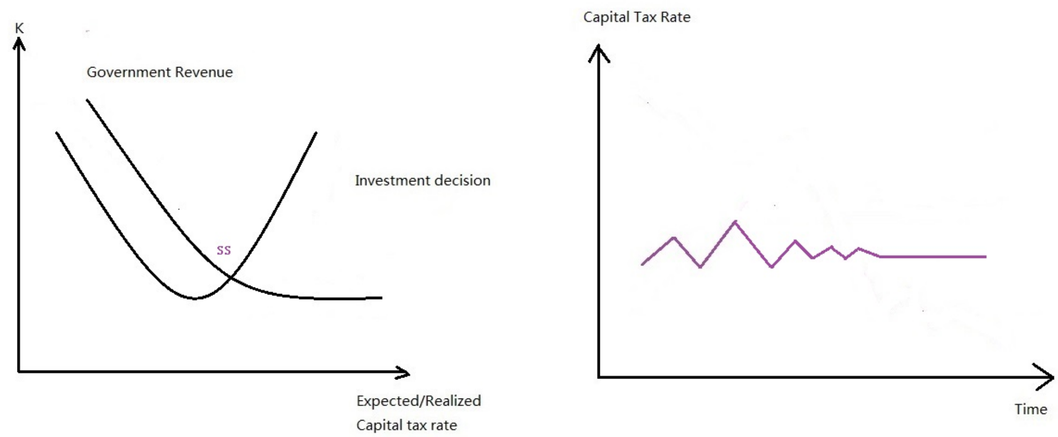

Figure 6: Pattern 3

Pattern 4

Similar to Pattern 3, investment decision is U-shaped but with higher substitution effect in the low range.

There are two equilibria in this economy, and neither of them is stable. Starting from any belief other than these two points, this economy converges to zero capital tax, coinciding with Ramsey's rule. Even if the economy starts with high belief, households invest significantly due to strong income effect, and this brings down the rate to the low range. Then the strong substitution effect comes into play and leads the economy to a decreasing tax rate and increasing capital level. This pattern is also beneficial to the economy.

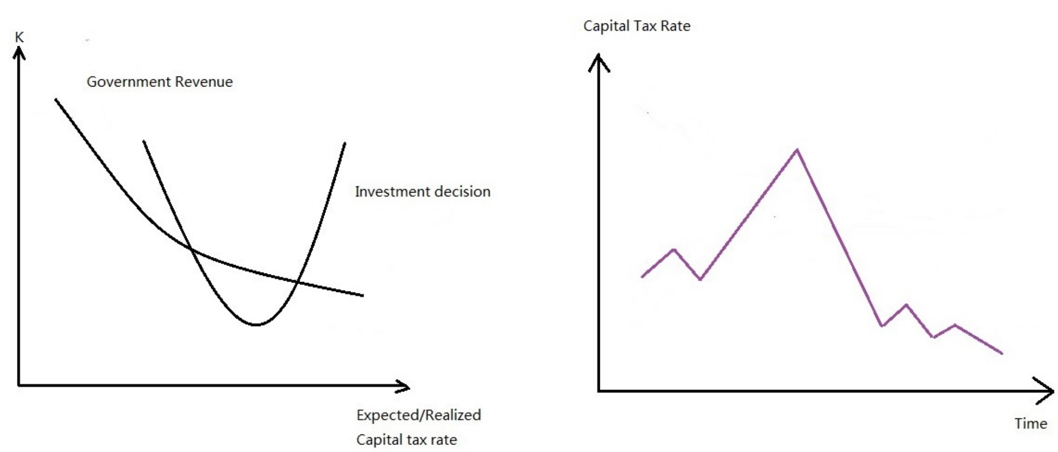

Figure 7: Pattern 4

Pattern 5

In the case with inverse U-shaped investment decision curve, there exist two equilibria and only one of them can be stable. Starting from any initial belief below a threshold level (denoted t in the figure), tax rates either oscillate around point O in Figure 8 or converge to it as a steady state. If the initial value is greater than t, tax rates diverge up to 1.

As income effect dominates at the low range, when tax rate increases, households have more incentive to invest, which in turn could bring down the tax rate. Thus tax rates alternate between high and low values, or ultimately converge to the steady state depending on the degree of income effect. At the high range, however, substitution effect dominates with households investing less with tax increase, which deteriorates the situation. Thus, tax rates diverge from a high value.

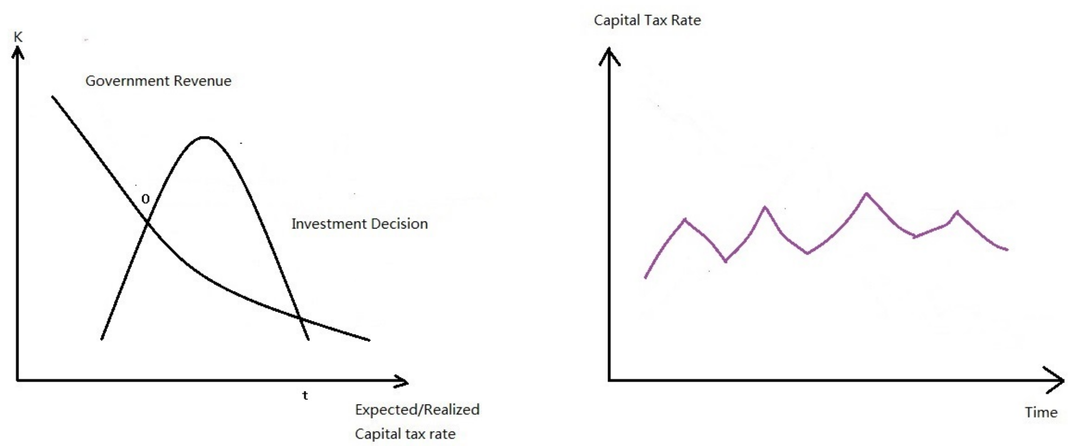

Figure 8: Pattern 5

Pattern 6

The case with cubic or higher degree polynomial investment curve is more complicated.

In the graph on the left of Figure 9, there exist multiple steady state equilibria but no stable one; and in the graph on the right, there exist two stable steady state equilibria and another unstable one.

The evolution path of capital tax rates can be various depending on the starting point and curvature of investment curve.

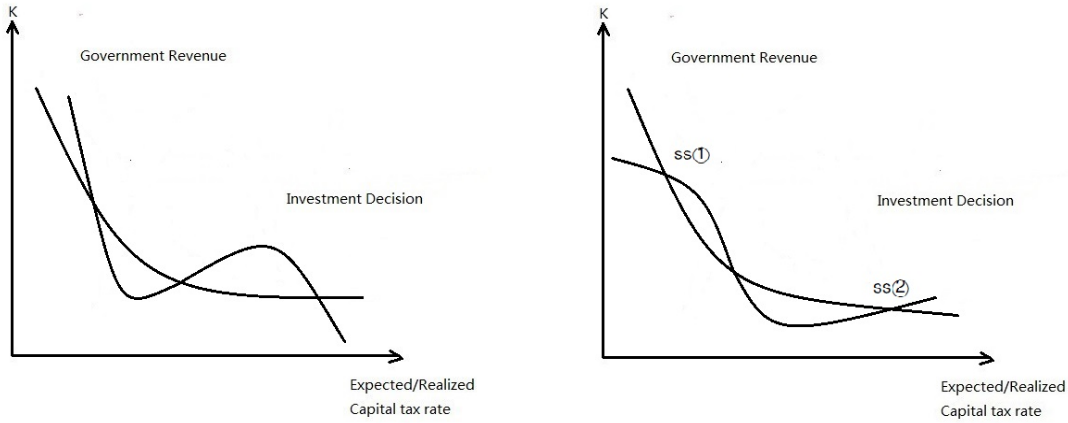

Figure 9: Pattern 6

4 Results

This section evaluates the model's performance to account for the empirical data observed empirically across 50 states.

4.1 Preliminary Test

The existence of multiple equilibria is tested. The two stable steady state equilibria in Figure 9, for instance, suggest that states with a low starting belief converge to an equilibrium with low rate while states with a high starting belief converge to an equilibrium with high rate.

In order to test this theory, I divided 51 states (DC included) into two subgroups by tax rates observed in year 1958. I assume $\theta^e_{1959} = \theta_{1958}$ as the starting belief of each state for year 1959. These two groups are named low-initial group and high-initial group, separated by the median value of initial beliefs.

I run AR(1) regression to obtain the steady state value. Then I use panel regression to test fixed effects across two groups, which is shown in Table 2.

Dependent Variable: TR?				
Method: Pooled Least Squares				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.006087	0.000831	7.321916	0.0000
LTR?	0.866378	0.010162	85.25481	0.0000
Fixed Effects (Cross)				
_HIGH--C	0.001787			
_LOW--C	-0.001787			
	Effects Specification			
Cross-section fixed (dummy variables)				
R-squared	0.755105	Mean dependent var	0.046272	
Adjusted R-squared	0.754909	S.D. dependent var	0.069102	
S.E. of regression	0.034210	Akaike info criterion	-3.911390	
Sum squared resid	2.917637	Schwarz criterion	-3.904392	
Log likelihood	4884.415	F-statistic	3843.436	
Durbin-Watson stat	1.837470	Prob(F-statistic)	0.000000	

Table 2: Fixed-Effect Test for Multiple Equilibria

I denote TR as capital tax rates and LTR as lagged capital tax rates. The hypothesis of different steady state levels across two groups is not rejected, and high-initial group does converge to a higher level of equilibrium rate. This preliminary test supports the model.

4.2 Simulation Results

I do simulations based on theoretical model to generate a sequence of capital tax rates for each state, and compare it with real data.

For adaptive expectation, I choose $\lambda = 0.9$ [8].

For the government revenue equation, I first normalize N_t=1 and set A_t=25 [9] for each state.

[8] This weight on new information is chosen such that the simulated result fits the data well. Comparative analysis on this weight is provided in later section.

[9] I choose the same A for states so as to fix the effects from A and isolate the effects of the model. Note that A captures factors more than Total Factor Productivity, such as composition of government tax revenue.

I calibrate the capital share α_t by 1-$\frac{TotalWage}{GSP}$ for each state and year.

As mentioned in previous section, the investment decision curve $k_t = f_i(\theta_t^e)$ for state i is chosen as the polynomial with the most significant regression result.

Feeding in $\theta_{1959}^e = \theta_{1958}$, the economy starts and a sequence of detrended capital chosen by households is generated, together with the sequence of realized capital tax rates.

Pick Alabama to interpret Case 1 of the theory.

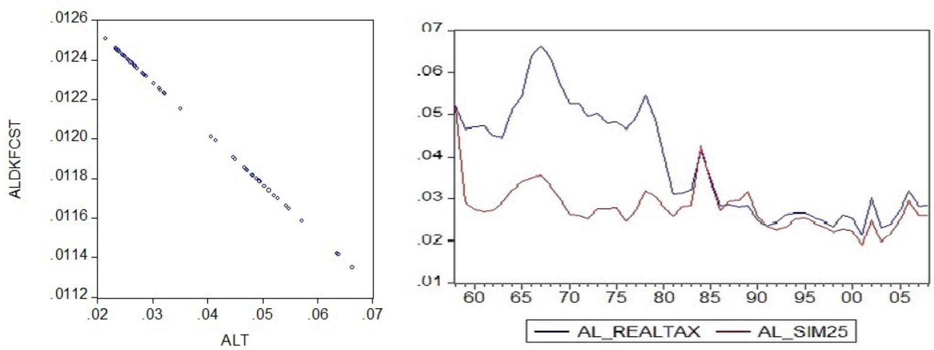

Figure 10: Simulation Results of Alabama

Graph on the left in Figure 10 shows that the investment decision in Alabama follows a decreasing pattern. Alabama starts with an initial tax rate equal to 0.05, which is relatively high. Then the pattern generated is a decreasing sequence, which matches the implication of the model. Moreover, the simulated data fits the real data after about 1980 [10].

Iowa represents the case with increasing investment decision curve, as shown in Figure 11. Following the implication of Inverse U-shaped case, tax rates will fluctuate around a steady state value if the economy does not start with a too high initial belief. Iowa's initial belief is 0.023, which is in the middle of tax range, and the simulation produces a fluctuating sequence.

[10] The generated pattern has small fluctuations over the decreasing trend rather than monotonely decreases, due to the fact that real data on government spending is not constant as in the model.

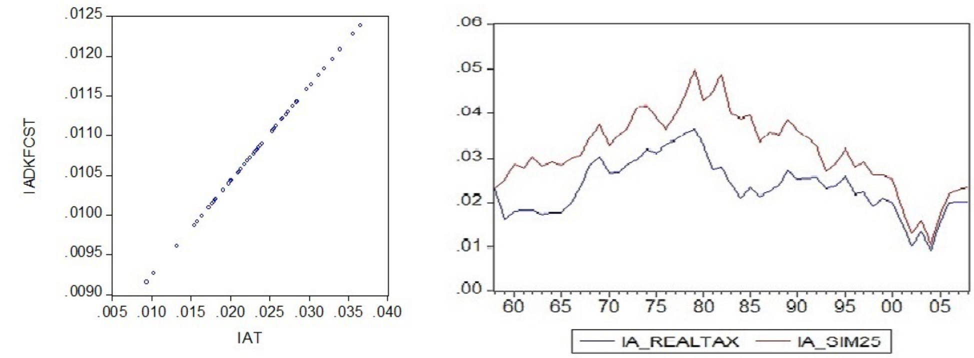

Figure 11: Simulation Results of Iowa

Similarly, New Mexico with an Inverse U-shaped investment curve also generates an oscillating path of capital tax rates, as is shown in Figure 12.

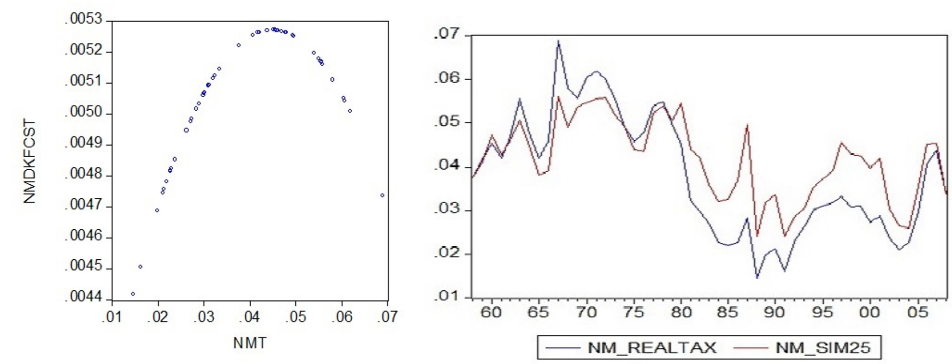

Figure 12: Simulation Results of New Mexico

Simulation result also produces a convergent-to-zero path, as suggested by the model. Virginia maintains a U-shaped investment curve and starts with a high belief at 0.06. The generated sequence fluctuates around a decreasing pattern and converges to 0, which also matches real data quantitatively.

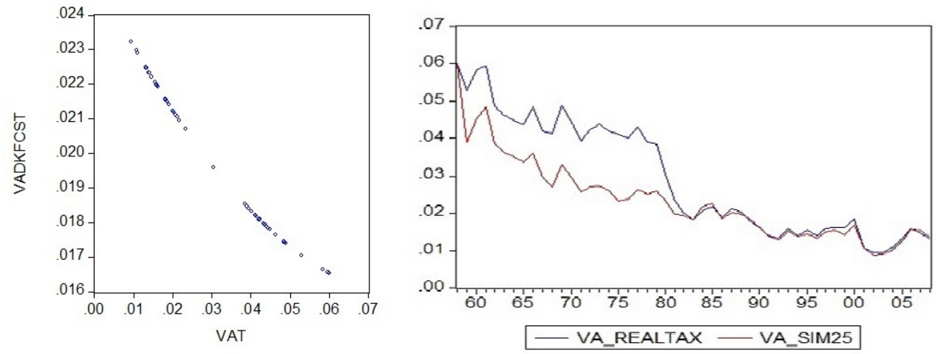

Figure 13: Simulation Results of Virginia

Illinois and Minnesota are two examples with cubic polynomial investment curve. Illinois starts with a low belief at 0.0069 and Minnesota starts with 0.056. Though the model suggests no uniform pattern in this more complicated case, the model can still generate data which captures the observed level and pattern in data, as depicted in Figure 14 and 15.

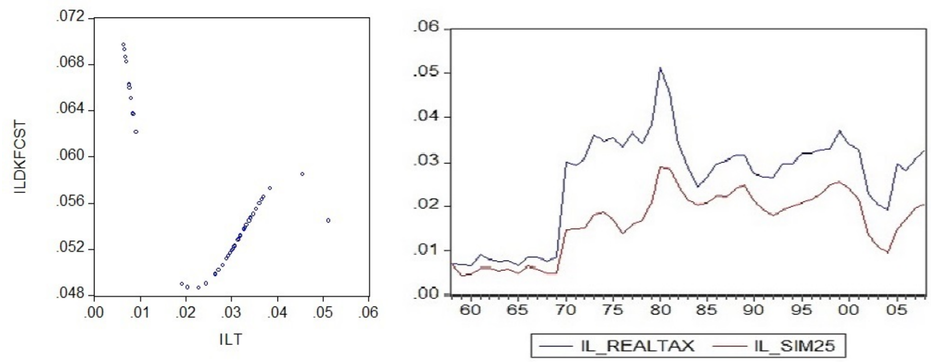

Figure 14: Simulation Results of Illinois

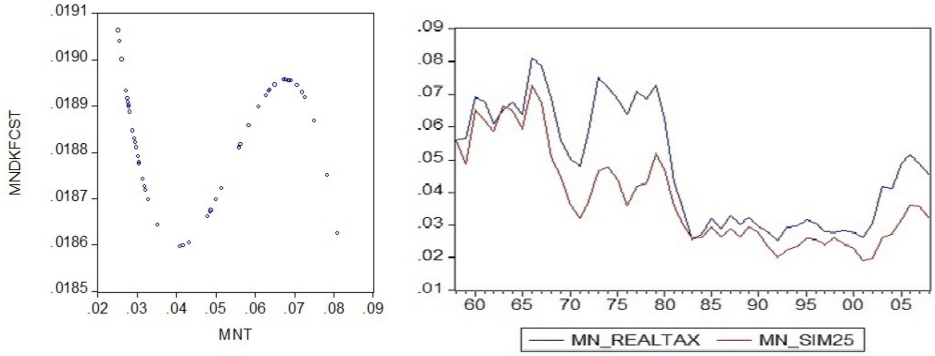

Figure 15: Simulation Results of Minnesota

4.3 Comparative Analysis

Comparative analysis is done regarding initial value and weight in Adaptive Expectation.

Firstly, I use Arizona data for comparative analysis on initial belief. Arizona follows a decreasing pattern of investment curve and according to theory, if there exist a positive steady state equilibrium which is what observed in data, then tax rates should increase monotonely toward it if starting from a low value.

The graph on the left of Figure 16 shows the simulation result as well as real data from the starting belief at a relatively high value, 0.079, which is the real tax rate in 1958. Both sequences display a decreasing pattern. The one on the right, however, feeds in a low initial belief to the economy. Consistent with the theory, tax rates increase to the steady state.

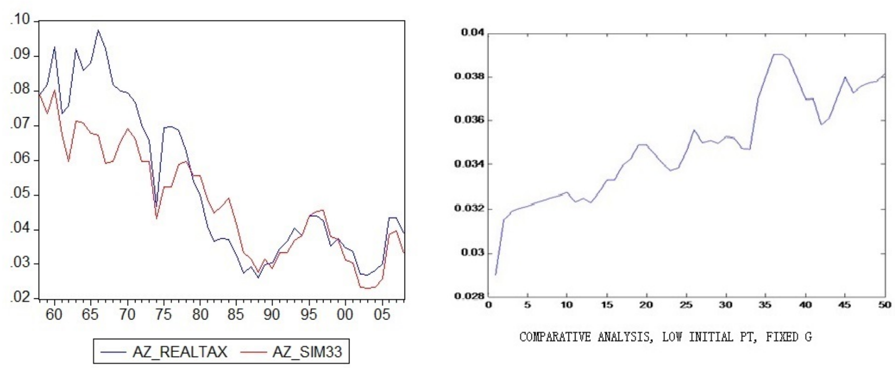

Figure 16: Comparative Analysis on initial belief

I use South Dakota to investigate the effects of belief updating process, which is shown in Figure 17. Recall that λ is the weight households place on new information to form expectation. The left figure presents the case with $\lambda = 1$, and the right one with $\lambda = 0.1$. Though tax rates follow the same pattern, tax rate sequence with $\lambda = 1$ has many spikes, while that under $\lambda = 0.1$ is more smoothed out.

With $\lambda = 1$, households completely rely on new information to update their belief, and with $\lambda = 0.1$, they gradually update their belief and the investment is smoothed out and so is the realized tax rate sequence. Putting a lower weight on previous expectation leads to more jumps of capital levels and thus of tax rates, when government spending level is not stable.

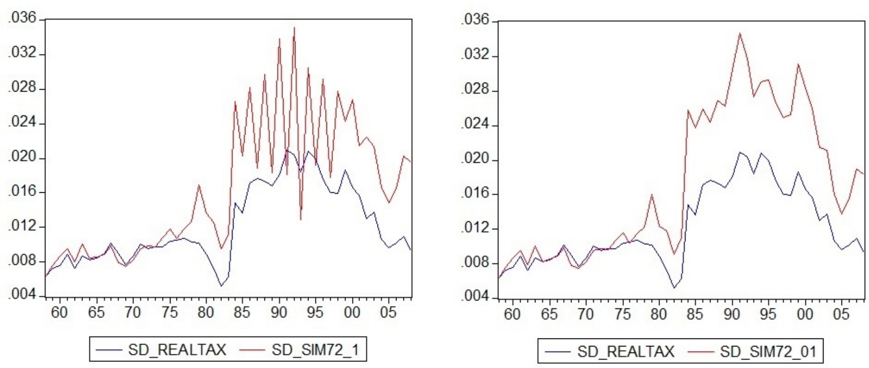

Figure 17: Comparative Analysis on belief updating process

5 Structural Model

This section bridges the gap between empirical model and real data with a structural model. The structural model targets the reduced-form investment decision curve, which is a key ingredient in the empirical model in the previous section.

5.1 Empirical Facts

States fall into different patterns of investment decision, as summarized in Appendix II. Each state maintains its own feature of geographic, economic and educational condition, and it is vital to discover the common traits in each group to explain states' different investment behaviors.
Two empirical facts on common traits are found[11]. Firstly, states with a decreasing investment curve are tested to have a higher average Gini coefficient than those with an increasing investment curve[12], with details in Appendix III.

[11] The differences in these two common traits are not significant in the groups of states with U-shaped or inverse U-shaped investment curves.

[12] The data on Gini coefficient in 2010 was obtained by U.S Census Bureau.

Households in a more equal economy tend to increase their investment as the expected tax rate increases, while households in a more polarized economy decrease aggregate investment as the expected tax rate increases.
Secondly, states with a decreasing investment curve are tested to have a higher level of education than those with an increasing investment curve[13], with details in Appendix III.
An overlapping generation model generates these two empirical facts.

5.2 Overlapping Generation Model with two types of households

Erosa and Gervais (2002) apply an Overlapping Generation (OLG) Model to present a reason for the nonzero capital tax if tax rates cannot be conditional on age. Garriga (2003) also theoretically analyzes the nonzero capital taxation under the framework of OLG model for a large class of preferences. In the finitely-lived household model, the distortion from capital taxation is much smaller than that in an infinitely-lived household model, and the consumption across the lifecycle of a household's life is not constant.
Following Swarbrik (2012), I introduce a two-agent two-period OLG Model with "wealthy" and "poor" households. Each household lives for two stages, young and old. Only "wealthy" households invest in capital when they are young and get capital income at the "old" stage. "Poor" households have no savings and consumes all their income each period. The "wealthy" makes up a portion of γ in the population[14] whilst the "poor" makes up the remaining $1-\gamma$ of the population. A superscript w denotes variables for the "wealthy" and p for the "poor".
Households obtain pension transfers from the government when they are old.
Households work and receive income from providing labor when they are young. "Wealthy" households can also invest in capital when they are young, and as they age, they have income sources from both capital returns and government pension transfers.
Assume utility function satisfies Inada conditions, and wealthy household chooses consumption for both periods as well as investment[15] to maximize

$$u(c_t^{y,w}) + \beta E_{\theta_{t+1}} u(c_{t+1}^{o,w}) \tag{6}$$

subject to the budget constraints for both periods:

$$c_t^{y,w} + k_{t+1} \leq w_t n_t \tag{7}$$

$$c_{t+1}^{o,w} \leq (1 + r_{t+1}(1-\theta_{t+1}))k_{t+1} + T_{t+1}^w \tag{8}$$

[13] I use Bachelor Degree Attainment from IMF as a measure to represent the level of education in each state.
[14] I assume $\gamma\epsilon(\frac{1}{2}, 1)$ to analyze the effect of equality in the economy, which ensures sufficient capital for production.
[15] To isolate the analyze on investment behavior, labor is normalized to $n_t = 1$ for both wealthy and poor for now.

Poor households without investing in capital can only consume with income from government pension transfers at the old stage. They only choose consumption in both periods to maximize:

$$u(c_t^{y,p}) + \beta E_{\theta_{t+1}} u(c_{t+1}^{o,p}) \tag{9}$$

subject to the budget constraints for both periods:

$$c_t^{y,p} \leq w_t n_t \tag{10}$$

$$c_{t+1}^{o,p} \leq T_{t+1}^p \tag{11}$$

Solving the problem of the "wealthy" gives the intertemporal Euler Equation with a complete global insurance company[16]:

$$u'(c_t^{y,w}) = \beta u'(c_{t+1}^{o,w}(\theta_{t+1}^e))(1 + r_{t+1}(1 - \theta_{t+1}^e)) \tag{12}$$

With a lack of commitment power from the government, households update their beliefs on future tax rates by Adaptive Expectation:

$$\theta_t^e = \lambda\theta_{t-1} + (1 - \lambda)\theta_{t-1}^e \tag{13}$$

A representative firm produces consumption goods with rented capital and employed labor with Cobb-Douglas production function:

$$Y_t = AK_t^\alpha N_t^{1-\alpha} \tag{14}$$

where aggregate capital $K_t = \gamma k_t$ and aggregate labor $N_t = 1$.
Rental rate and wage rate are:

$$r_t = A\alpha K_t^{\alpha-1} N_t^{1-\alpha} - \delta \tag{15}$$

$$w_t = A(1 - \alpha)K_t^\alpha N_t^{-\alpha} \tag{16}$$

Government's budget constraint determines the capital tax rate for each period t, given G_t, T_t^w and T_t^p.

$$G_t + \gamma T_t^w + (1 - \gamma)T_t^p = K_t r_t \theta_t \tag{17}$$

The free parameters A and γ in the model capture the level of education and degree of inequality in the economy.
Moretti (2004) provides evidence that human capital is positively correlated with productivity due to externalities. He calculates the fraction of college-educated workers among all to index the level of human capital, which is consistent with

[16] The completeness in the global insurance company enables households to fully insure their consumption against capital tax uncertainty.

the empirical data of Bachelor Degree Attainment I use for each state. Moretti (2004) finds that with human capital spillover, cities with a larger stock of human capital are more productive than those with a smaller stock. Supported by Moretti (2004)'s work, it is legitimate to capture A in the model by Bachelor Degree Attainment in the state.
As for how γ in the model captures the degree of equality in an economy, Gini coefficient is calculated with the illustration of Figure 18.

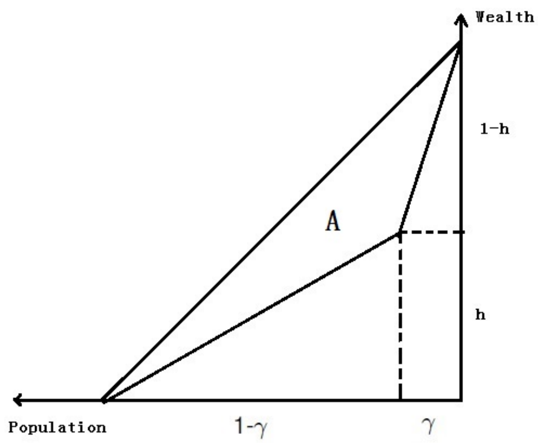

Figure 18: Gini Coefficient

In the economy with two types of households holding wealth w_1, w_2 respectively, where $w_1 < w_2$. Gini coefficient (G) equals $\frac{A}{\frac{1}{2}} = 1 - \gamma - h$, where $h = \frac{(1-\gamma)w_1}{(1-\gamma)w_1+\gamma w_2}$. Plugging in gives $G = \frac{w_2-w_1}{\frac{w_1}{\gamma}+\frac{w_2}{1-\gamma}}$. $\frac{w_1}{\gamma} + \frac{w_2}{1-\gamma}$ decreases in $\gamma \in (0, \gamma^*)$ and increases in $\gamma \in (\gamma^*, 1)$, where $\gamma^* < \frac{1}{2}$. Thus the economy is most unequal at γ^* and becomes more equal as γ approaches to either 0 or 1.
Now take differentiation on Intertemporal Euler Equation (11) with respect to θ^e_{t+1} and k_{t+1} to obtain the curve of investment decision.

Thus,

$$\frac{dk_{t+1}}{d\theta^e_{t+1}} = \frac{\beta r_{t+1}\Psi}{\beta\Psi(1-\theta^e_{t+1})\frac{dr_{t+1}}{dk_{t+1}} + \beta u''[(1+r_{t+1}(1-\theta^e_{t+1}))k_{t+1}+T^w_{t+1}](1+r_{t+1}(1-\theta^e_{t+1}))^2 + u''(w_t-k_{t+1})} \tag{18}$$

where

$$\Psi = u''[(1+r_{t+1}(1-\theta^e_{t+1}))k_{t+1}+T^w_{t+1}](1+r_{t+1}(1-\theta^e_{t+1}))k_{t+1}+u'[(1+r_{t+1}(1-\theta^e_{t+1}))k_{t+1}+T^w_{t+1}]$$

and $\frac{dr_{t+1}}{dk_{t+1}} = A\alpha(\alpha-1)\gamma^{\alpha-1}k_{t+1}^{\alpha-2} < 0$.
Suppose $\Psi > 0$, then the denominator is negative and the investment decision curve is downward sloping.
As A decreases and γ increases, Ψ decreases with a utility function satisfying certain conditions characterized as follows.

Condition 5 *The utility function holds the following condition:*
u(c) has the Elasticity of Intertemporal Substitution, $-\frac{u'(c)}{u''(c)c}$*, below one at lower range of c and above one at higher range of c such that* $u''(c)c + u'(c) < 0$ *at low c and* > 0 *at high c.*

Ψ can decrease to be negative. With a smaller A and a bigger γ, $\mid \frac{dr_{t+1}}{dk_{t+1}} \mid$ decreases. Hence, with $\Psi < 0$, the denominator can remain negative and the total effect is positive, which implies that the investment decision curve is upward sloping.
The intuition is as follows: A lower TFP value decreases households' income at each period. With a strong consumption smoothing effect at a low consumption value as suggested by Condition 1, the income effect becomes stronger when the income is lower. Moreover, a higher γ increases the portion of wealthy households who invest in capital which in turn reduces the return of aggregate investment and alleviates the substitution effect. Thus in a more equalized economy with more investors and a lower productivity level, income effect dominates substitution effect and the investment decision curve is sloped upwards.

6 Conclusion

This paper provides an alternative explanation for the possibility of nonzero capital taxation in the economy. Government's lack of commitment power forces households to form their own expectations on tax rates. Furthermore, the balanced budget constraint disenables the government to freely set the capital tax rates in order to alter households' belief. Consequently, capital tax rates are history-dependent.
The pattern of historic capital rate development depends on two factors: initial belief on the capital tax rate and state-specific investment behavior. The overall education level as well as the degree of equality in the economy determines the investment decision curve for each state, according to empirical observations. An overlapping generation model with two heterogeneous agents can produce this result as long as the utility function satisfies certain conditions.
A more equalized economy with a lower productivity level increases income effect and decreases substitution effect, which generates an increasing investment decision curve.
For some cases of this theory, the economy will converge to a zero capital tax rate, which coincides with Ramsey's result. With an elastic investment curve

and a low enough initial tax belief, capital tax rates converge to zero in the long run.
This paper simulates capital tax rate patterns which match the real data. The future study is possibly to extend the existing model to further rationalize the U-shaped and inverse U-shaped investment decision curves.

References

[1] Aiyagari, R., "Optimal Capital Income Taxation with Incomplete Markets, Borrowing Constraints, and Constant Discounting," *Journal of Political Economy*, 103, 1995: 158-75.

[2] Bonham, C., and R. Cohen, "To Aggregate, Pool, or Neither: Testing the Rational Expectations hypothesis Using Survey Data," *Journal of Business and Economic Statistics*, 19,2001, 278-291.

[3] Bonham, C., and D. Dacy, "In Search of a Strictly Rational Forecast," *Review of Economics and Statistics*, 73, 1991,245-253.

[4] Campbell, John Y. and Shiller, Robert J., "Cointegration and Tests of Present Value Models," *Journal of Political Economy*, 95, 1987: 1062-1088.

[5] Chamley, C., "Optimal Taxation of Capital Income in General Equilibrium with Infinite Lives," *Econometrica*, 54, 1986: 607-622.

[6] Croushore, D., "The Livingston Survey: Still Useful After All These Years," *Federal Reserve Bank of Philadelphia Business Review*,1997 March/April, 15-26.

[7] Gregory C. Chow, "Rational versus Adaptive Expectations in Present Value Models," *Review of Economics and Statistics*,71(3), 1989: 376-384.

[8] Gregory C. Chow, "Usefulness of Adaptive and Rational Expectations in Economics," *C*EPS Working Paper No. 221, 2011.

[9] Diamond P.A.and Spinnewijn J., "Capital Income Taxes with Heterogeneous Discount Rates," *American Economic Journal: Economic Policy, American Economic Association*, vol. 3(4), 2011, pages 52-76, November.

[10] Erosa, A. and M. Gervais, "Optimal Fiscal Policy in Life-Cycle Economies," *Journal of Economic Theory*, 105, 2002: 338-369.

[11] Fama, Eugene F. and French, Kenneth R., "Permanent and Temporary Components of Stock Prices," *Journal of Political Economy*, 96, 1988: 246-273.

[12] Frankel, J., and K. Froot, "Using Survey Data to Test Standard Propositions Regarding Exchange Rate Expectations," *American Economic Review*, 77, 1987b, 133-153.

[13] Frankel, J., and K. Froot, “Chartists, Fundamentalists and the Demand for Dollars,” *i*n A.S. Courakis and M. P. Taylor, ed.,Private Behaviour and Government Policy in Interdependent Economies, Oxford University Press, Oxford, pp.73-126.

[14] Friedman, B., “Survey Evidence on the ’Rationality’ of Interest Rate Expectations,” *Journal of Monetary Economics*, 6, 1990, 453-465.

[15] Froot, K. A., “New Hope for the Expectations Hypothesis of the Term Structure of Interest Rates, ” *Journal of Finance*, 44, 1989, 283-305.

[16] Garriga, C., “Optimal Fiscal policy in overlapping generations models,” *W*orking Paper, Florida State University.

[17] Garofalo G.A. and Yamarik S., “Regional Convergence: Evidence From A New State-By-State Capital Stock Series,” *The Review of Economics and Statistics*, MIT Press, vol. 84(2), 2002, pages 316-323.

[18] Hartman, David G., “Tax policy and foreign direct investment,” 1985, *Journal of Public Economics,* 26, 107-21.

[19] R. Glenn Hubbard and Kenneth L. Judd, “Liquidity Constraints, Fiscal Policy, and Consumption,” Papers on Economic Activity, Economic Studies Program, The Brookings Institution, vol. 17(1), pages 1-60.

[20] Jeong, J., and G. S. Maddala,“Testing the Rationality of Survey Data Using the Weighted Double-bootstrapped Method of Moments,” *Review of Economics and Statistics*, 78, 1996 296-302.

[21] Judd, K.L., “Redistributive Taxation in a Simple Perfect Foresight Model,” 1985, *Journal of Public Economics*, 28, 59-83.

[22] Brian Knight, “State capital taxes and the location of investment: empirical lessons from theoretical models of tax competition,” Finance and Economics Discussion Series 2002, Board of Governors of the Federal Reserve System (U.S.).

[23] Koehne, Sebastian and Kuhn, Moritz, “Optimal capital taxation for timenonseparable preferences,” MPRA Paper 45203, University Library of Munich, Germany.

[24] Markiewicz, A. and Pick, A., “Adaptive Learning and Survey Data,” *C*DMA Working Paper Series 201305, Centre for Dynamic Macroeconomic Analysis.

[25] Moretti, E., “Workers’ Education, Spillover and Productivity: Evidence from Plant-Level Production Functions„” *American Economic Review* 94(3),2004, 656-690.

[26] Pomerleau K., “The High Burden of State and Federal Capital Gains Tax Rates,” *T*he Tax Foundation, 2014.

[27] Poterba, James M. and Summers, Lawrence H., “Mean Reversion in Stock Prices: Evidence and Implications,” NBER Working Paper, 1987.

[28] Ramsey, F.P., “A Contribution to the Theory of Taxation,” *Economic Journal*, Vol. 37, No 145, 1927, 47-61.

[29] Swarbrick J., “Optimal social policy in a DSGE model with heterogeneous agents ,” Master thesis, 37-62.

[30] Yamarik, S., “State-Level Capital And Investment: Updates And Implications,” *Contemporary Economic Policy*, Western Economic Association International, vol. 31(1),2013, pages 62-72, 01.

[31] Young, Kan H., “The effects of taxes and rates of return on foreign direct investment in the United States,” *National Tax Journal*, 41 (1),1988: 109-21.

Appendix I

Table 2: Initial Belief

Initial Belief	States
0.001-0.01	Illinois; South Dakota; West Virginia
0.01-0.02	Florida; Maine; New Jersey; Ohio
0.02-0.03	Delaware; Iowa; Indiana; Kansas; Massachusetts; Michigan; Missouri; North Dakota; New Hampshire; Nevada; Texas
0.03-0.04	Georgia; Hawaii; New Mexico; Oklahoma;
0.04-0.05	Connecticut; Maryland; Utah;
	Arkansas; Montana; Rhode Island; Tennessee; Vermont; Washington;
0.05-0.06	Alabama; Alaska; California; Colorado; Idaho; Louisiana; Minnesota; Nebraska; New York;
0.06-0.07	Oregon; Pennsylvania; South Carolina; Virginia;
0.07-0.08	Arizona; Kentucky; Mississippi; Wyoming
0.08-0.09	North Carolina; Wisconsin
0.1-	DC

Appendix II

Table 3: Sensitivity Pattern

Sensitivity Pattern	States
Decreasing	Alabama; Arizona; Arkansas; Colorado; DC; Indiana; Massachusetts; Maryland; North Carolina; South Dakota; Utah; Oregon;
Increasing	Iowa; Idaho; Kansas; Kentucky; North Dakota; Nebraska; Rhode Island; Wisconsin; Oklahoma;
U-shape	California; Georgia; Maryland; Michigan; New Jersey; Nevada; South Carolina; Tennessee; Texas; Virginia; West Virginia; Wyoming;
Inverse U-shape	Alaska; Delaware; Florida; Hawaii; Louisiana; Missouri; Mississippi; Montana; New Mexico; Pennsylvania;
Cubic or higher	Connecticut; Illinois; Minnesota; New Hampshire; New York; Ohio; Vermont; Washington

Appendix III

Table 4: Tests for Equality of Means between Series: Gini Coefficient

Method	df	Value	Probability
t-test	19	1.536094	0.1410
Anova F-statistic	(1, 19)	2.359585	0.1410
Category Statistics			
Variable	Count	Mean	Std. Dev.
DECR	12	0.458833	0.027643
INCR	9	0.443000	0.015716

Table 5: Tests for Equality of Means between Series: Gini Coefficient (outlier Utah excluded)

Method	df	Value	Probability
t-test	18	1.974393	0.0639
Anova F-statistic	(1, 18)	3.898228	0.0639
Category Statistics			
Variable	Count	Mean	Std. Dev.
DECR	11	0.462455	0.025835
INCR	9	0.453700	0.023535

Table 6: Tests for Equality of Means between Series: Bachelor Degree Attainment

Method	df	Value	Probability
t-test	19	1.400530	0.1775
Anova F-statistic	(1, 19)	1.961485	0.1775
Category Statistics			
Variable	Count	Mean	Std. Dev.
DECR	12	30.26667	8.746151
INCR	9	25.96667	3.155551

Table 7: Tests for Equality of Means between Series: Bachelor Degree Attainment (outlier Arkansas excluded)

Method	df	Value	Probability
t-test	18	1.761973	0.0950
Anova F-statistic	(1, 18)	3.104550	0.0950
Category Statistics			
Variable	Count	Mean	Std. Dev.
DECR	11	31.23636	8.469507
INCR	9	25.96667	3.155551